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Acceleration radiation in a compact space

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Abstract. We study the response of a uniformly accelerated model particle detector in a spacetime with compact spatial sections. The basic thermal character of the response re-emerges, in spite of the fact that the spacetime does not possess event horizons. Our model also permits a study of detector response to twisted field states.

1. Introduction

The discovery that a uniformly accelerated observer perceives thermal radiation (Davies 1975, Unruh 1976) has proved an important probe of quantum field theory in spacetime with non-trivial causal and/or geometrical structure, and provides a useful analogy to the black hole radiance phenomenon (Hawking 1975). The intimate association between the thermal character of this 'acceleration radiation' and the presence of event horizons in the frame of the accelerated observer has often been stressed (Gibbons and Hawking 1977, Sciama 1979). This had led to the general assumption that thermal acceleration radiation is best regarded as a global phenomenon which owes its origin to the causal structure of the so-called Rindler wedge (figure 1). On the other hand, Sanchez (1979) has constructed accelerated frames without event horizons where particle production takes place with thermal and non-thermal distributions.

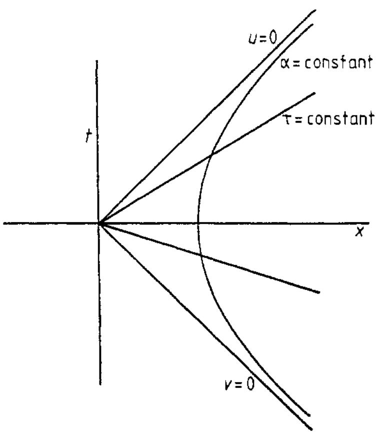

Figure 1. The Rindler wedge is the right-hand region between null lines $u=0$ (future event horizon) and $v=0$ (past event horizon). The detector trajectory is represented by the hyperbola which is the world-line of a particle moving with constant acceleration (α = constant). The line τ = constant is a line of constant proper time.

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In this paper we study the response of a model particle detector which is uniformly accelerated in flat two-dimensional spacetime with $S^1 \times R^1$ topology. This spacetime has a compact (S^1) spatial section of length L, but is locally identical to Minkowski space. Thus, locally the detector is equivalent to the Rindler case (see e.g. Birrell and Davies 1982, § 4.5) but globally the situation is completely different (see figure 2). Specifically the spacetime no longer possesses event horizons in the frame of the accelerated detector. This is easily understood. The asymptotic null ray to which the detector's world-line tends at infinite proper-time winds round and round the spacetime 'cylinder'. Thus there exist no events which lie entirely 'above' this null ray (i.e. no events from which all future-directed null rays fail to intersect the asymptotic null ray). All events therefore lie to the past of a portion of the detector's world-line.

By constrast, in the Rindler case (figure 1), some information about the quantum state in the region to the left of the diagram will be foreover inaccessible to the accelerated detector. This forfeiture of information is neatly identified with the entropy associated with the thermal nature of the Rindler detector's response. The question then arises as to what extent the thermal acceleration radiation can be identified with the existence of the event horizon. Is it a global (causal) or a local feature? We find that in the absence of the horizon, a uniformly accelerated detector still responds as though immersed in a time-independent bath of radiation with a Planck spectrum for an untwisted field. However, with a twisted field, the constant Planckian flux is accompanied by a time-dependent transient term.

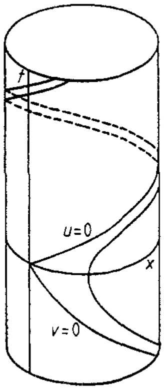

Figure 2. For the $S^1 \times R^1$ spacetime, the uniformly accelerating trajectory winds around the 'cylinder'. This winding results in all regions of the spacetime being causally connected to the trajectory of the detector. Hence there is no event horizon.

2. Detector response

Our treatment is a direct adaptation of the original calculation of Unruh (1976) though we prefer to use the detector model of DeWitt (1979).

The first-order transition amplitude for excitation of the DeWitt detector is (see DeWitt 1979 or Birrell and Davies 1982)

$$\mathrm{i}c\langle E|m(0)|E_0\rangle \int_{-\infty}^{\infty} \mathrm{d}\tau \exp[\mathrm{i}(E-E_0)\tau]\langle\psi|\phi[x^\mu(\tau)]|0\rangle \tag{2.1}$$

where $m(\tau)$ is the detector's monopole moment, τ the proper time, E_0 the ground state energy of the detector, E the excited state energy, c a (small) coupling constant, and $x^\mu(\tau)$ the trajectory of the detector. The quantum state $|0\rangle$ is the vacuum state of the massless scalar field as determined by an inertial observer, which in this case is the 'Casimir' vacuum (see Birrell and Davies 1982, § 4). This vacuum state reduces to the usual Minkowski vacuum for $L \to \infty$. The state $|\psi\rangle$ is an excited state, which in first-order perturbation theory only gives a non-vanishing matrix in (2.1) if it is a one-particle state.

Differentiating (2.1), the transition rate (probability per unit detector time τ) at time τ_0 can be written in the form

$$c^2|\langle E|m(0)|E_0\rangle|^2 \int_{-\infty}^{\infty} \mathrm{d}\eta\, \mathrm{e}^{-\mathrm{i}\Delta E\eta}[G^+(x(\eta+\tau_0), x(\tau_0))\theta(-\eta) + G^+(x(\tau_0), x(\tau_0-\eta))\theta(\eta)] \tag{2.2}$$

where $\Delta E = E - E_0$ and $G^+(x(\tau), x(\tau'))$ is the Wightman Green function for the vacuum state $|0\rangle$.

The scalar field ψ may be expanded in terms of discrete modes

$$u_k = (2L\omega)^{-1/2} \exp[\mathrm{i}(kx - \omega t)], \qquad \omega = |k|, \tag{2.3}$$

$$k = 2\pi m/L, \qquad m = \pm 1, \pm 2 \ldots. \tag{2.4}$$

Introducing null coordinates $u = t - x$, $v = t + x$, the modes may be split into those which are right moving $\vec{u}_k$ and left moving $\overleftarrow{u}_k$:

$$\left.\begin{aligned} \vec{u}_k &= \exp(-2\pi \mathrm{i} nu/L)/(4\pi n)^{1/2} \\ \overleftarrow{u}_k &= \exp(-2\pi \mathrm{i} nv/L)/(4\pi n)^{1/2} \end{aligned}\right\} n = 1, 2, 3 \ldots.$$

whereupon we may write

$$\begin{aligned} \langle \vec{1}_k|\phi[x]|0\rangle &= \exp(2\pi \mathrm{i} nu/L)/(4\pi n)^{1/2} \\ \langle \overleftarrow{1}_k|\phi[x]|0\rangle &= \exp(2\pi \mathrm{i} nv/L)/(4\pi n)^{1/2} \end{aligned} \tag{2.5}$$

where $|\vec{1}_k\rangle$ represents a one-particle right-moving state and $|\overleftarrow{1}_k\rangle$ a corresponding left-moving state. The world-line of a uniformly accelerating detector in the $S^1 \times R^1$ spacetime is described piecewise by

$$x = \alpha \cosh \bar{\tau} - mL \tag{2.6}$$

$$t = \alpha \sinh \bar{\tau} \tag{2.7}$$

where $\bar{\tau} = \alpha^{-1}\tau$, α being the proper acceleration, τ the proper time and m an integer with the interpretation of 'winding number' (i.e. the number of times the detector has orbited the cylinder). Note that the compactification of the space breaks the Lorentz symmetry and introduces a privileged frame.

We choose the winding number $m = 0$ to include the portion of the trajectory (2.6) and (2.7) at which the detector is at rest in the privileged frame ($\tau = 0$). So positive values of m correspond to $\tau > 0$ and negative to $\tau < 0$. The integral in (2.1) decomposes as follows:

$$\int_{-\infty}^{\infty} \mathrm{d}\tau \to \alpha \sum_{m=0}^{\infty} \left\{ \int_{\bar{\tau}_m}^{\bar{\tau}_{m+1}} \mathrm{d}\bar{\tau} + \int_{-\bar{\tau}_{m+1}}^{-\bar{\tau}_m} \mathrm{d}\bar{\tau} \right\} = \alpha \sum_{m=-\infty}^{\infty} \int_{\bar{\tau}_m}^{\bar{\tau}_{m+1}} \mathrm{d}\bar{\tau} \tag{2.8}$$

where $\bar{\tau}_m = \cosh^{-1}[mL/\alpha]$, $m>0$ and $\bar{\tau}_m = -\cosh^{-1}[mL/\alpha]$, $m<0$. Substituting (2.6) and (2.7) into (2.5) which is then placed into (2.1) and using (2.8), we obtain for the contributions to the transition amplitude from modes $|\vec{1}_k\rangle$ and $|\overleftarrow{1}_k\rangle$

$$\text{constant}\times \sum_{m=-\infty}^{\infty} \int_{\bar{\tau}_m}^{\bar{\tau}_{m+1}} \mathrm{d}\bar{\tau}\{\exp[(2\pi i n\alpha/L)(\sinh\bar{\tau}-\cosh\bar{\tau})+2\pi i n m]\exp(i\alpha\Delta E\bar{\tau})\} \tag{2.9}$$

and

$$\text{constant}\times \sum_{m=-\infty}^{\infty} \int_{\bar{\tau}_m}^{\bar{\tau}_{m+1}} \mathrm{d}\bar{\tau}\{\exp[(2\pi i n\alpha/L)(\sinh\bar{\tau}+\cosh\bar{\tau})-2\pi i n m]\exp(i\alpha\Delta E\bar{\tau})\}$$

respectively.

To evaluate (2.9) we split the sums into two parts: the $m=0$ strip and $|m|>0$. The $m=0$ part gives for the $|\vec{1}_k\rangle$ and $|\overleftarrow{1}_k\rangle$ contributions

$$\int_{-\cosh^{-1}(L/\alpha)}^{\cosh^{-1}(L/\alpha)} \mathrm{d}\bar{\tau}\,[\exp(-2\pi i n\alpha\,\mathrm{e}^{-\bar{\tau}}/L)\exp(i\alpha\Delta E\bar{\tau})]$$

and (2.10)

$$\int_{-\cosh^{-1}(L/\alpha)}^{\cosh^{-1}(L/\alpha)} \mathrm{d}\bar{\tau}\,[\exp(2\pi i n\alpha\,\mathrm{e}^{\bar{\tau}}/L)\exp(i\alpha\Delta E\bar{\tau})]$$

yielding

$$a^{i\Delta E\alpha}\,\mathrm{e}^{-\pi\Delta E\alpha/2}[\Gamma(-i\Delta E\alpha, i\alpha\exp(-\cosh^{-1}(L/\alpha)))-\Gamma(-i\Delta E\alpha, i\alpha\exp(\cosh^{-1}(L/\alpha)))] \tag{2.11}$$

and its complex conjugate respectively where $a=2\pi n\alpha/L$. The $|m|>0$ part yields similarly

$$\mathrm{e}^{-\pi\Delta E\alpha/2}\sum_{m=1}^{\infty}\{[\Gamma(-i\Delta E\alpha, iaf(m))-\Gamma(-i\Delta E\alpha, ia/f(m)) \\ -\Gamma(-i\Delta E\alpha, iaf(m+1))+\Gamma(-i\Delta E\alpha, ia/f(m+1))]a^{i\Delta E\alpha}\} \tag{2.12}$$

and its complex conjugate where $f(n)=\exp[\cosh^{-1}(nL/\alpha)]$.

The sums are readily evaluated by noting that the first two terms in the summand cancel, for each m, the second two terms for the previous value of m. Evaluating the infinite sum using $\Sigma_{m=1}^{\infty} = \lim_{N\to\infty}\Sigma_{m=1}^{N}$ and noting that the remaining $m=1$ terms are exactly cancelled by the corresponding $m=0$ terms, we find (2.9) reduces to

$$\mathrm{e}^{-\pi\Delta E\alpha/2}\lim_{m\to\infty}[\Gamma(-i\Delta E\alpha, ia/f(m+1))-\Gamma(-i\Delta E\alpha, iaf(m+1))]a^{i\Delta E\alpha}$$

and its conjugate respectively.

Using the integral representation of the Γ-function we analytically continue into the complex plane as shown in figure 3. Taking the limit gives for the contributions to the transition amplitude

$$ic\alpha\langle E|m(0)|E_0\rangle\,\mathrm{e}^{\pi\Delta E\alpha/2}[(2\pi n\alpha/L)^{i\Delta E\alpha}\Gamma(-i\Delta E\alpha)]/(4\pi n)^{1/2}$$

and (2.13)

$$ic\alpha\langle E|m(0)|E_0\rangle\,\mathrm{e}^{-\pi\Delta E\alpha/2}[(2\pi n\alpha/L)^{-i\Delta E\alpha}\Gamma(i\Delta E\alpha)]/(4\pi n)^{1/2}.$$

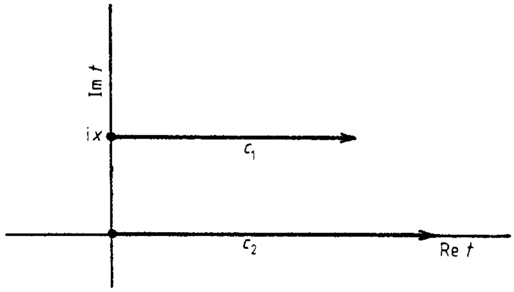

Figure 3. Using the integral representation of the incomplete gamma function given in the text, the contour C_2 represents a complete gamma function and the contour C_1 represents the analytical continuation of the incomplete Γ-function into the complex t-plane, used to evaluate $\Gamma(a, \mathrm{i}x)$ where $x \in \mathbb{R}$.

From this we compute the transition probability for excitation of the detector to energy level E by adding the sums of the squared moduli of (2.13) over all n, yielding

$$\alpha^2 c^2 |\langle E|m(0)|E_0\rangle|^2 \, \mathrm{e}^{-\pi\Delta E\alpha} \sum_{n=1}^{\infty} |\Gamma(\mathrm{i}\Delta E\alpha)|^2/n. \tag{2.14}$$

Using the identity $|\Gamma(\mathrm{i}x)|^2 = \pi/x \sinh(\pi x)$, expression (2.14) is found to be

$$c^2|\langle E|m(0)|E_0\rangle|^2[(E-E_0)^{-1}(\exp(2\pi\alpha\Delta E)-1)^{-1}] \sum_{n=1}^{\infty} (\alpha/n) \tag{2.15}$$

which yields a constant transition rate

$$c^2|\langle E|m(0)|E_0\rangle|^2/[(E-E_0)(\exp(2\pi\alpha(E-E_0))-1)].$$

The total response as given in (2.15) is divergent. This is expected because the detector is accelerating (and responding) for all time. Dividing out the $\Sigma_n\ (\alpha/n)$ yields a constant finite transition rate which is identical to the $\mathbb{R}^2$ Rindler case (Birrell and Davies 1982). The energy spectrum is manifestly Planckian. However, we note that the detector's response is a *continuous* (in ΔE) Planckian spectrum, whereas its response when at rest in $S^1 \times R^1$ (in the priveleged frame) to a thermal state at temperature $T = 2\pi/\alpha$ would involve a Planckian envolope to a *discrete* spectrum.

In summary then, we find a thermal response; thus in spite of the absence of event horizons the Rindler character of the response re-emerges (see also Sanchez 1981).

3. Twisted fields

The existence of a non-trivial spacetime topology in the model allows the possibility of antiperiodic boundary conditions, or 'twisted' field configurations (Isham 1978). Interest then attaches to the response of the accelerated detector to the twisted vacuum state. In particular, will an accelerated detector differentiate between twisted and untwisted vacua?

The antiperiodic boundary conditions may be achieved by replacing (2.3) by

$$k = 2\pi(m + \tfrac{1}{2})/L. \tag{3.1}$$

The calculation of the detector's response then proceeds along the lines of the untwisted case given in § 2. However, in this case the transition amplitude summations, analogous to (2.12), cannot be evaluated by simply noting the cancellation of successive terms, due to an alternating sign arising from the antiperiodicity. Nevertheless, we can perform an approximate calculation of the transition rate for $L \gg \alpha$.

For the twisted case, we have in place of (2.5)

$$\left.\begin{array}{l}\langle \vec{1}_k|\phi[x]|0\rangle = \exp[\pi i u(2n+1)/L]/[2\pi(2n+1)]^{1/2} \\ \langle \bar{1}_k|\phi[x]|0\rangle = \exp[\pi i v(2n+1)/L]/[2\pi(2n+1)]^{1/2}\end{array}\right\} n = 0, 1, 2, \ldots . \qquad (3.2)$$

Using (2.6), (2.7) and (2.8) we obtain contributions to the transition amplitude

$$\text{constant} \times \sum_{m=-\infty}^{\infty} \int_{\bar{\tau}_m}^{\bar{\tau}_{m+1}} \mathrm{d}\bar{\tau}\, (-1)^m [\exp(-\pi \mathrm{i}(2n+1)\alpha\, \mathrm{e}^{-\bar{\tau}}/L) \exp(\mathrm{i}\alpha \Delta E \bar{\tau})]$$

and (3.3)

$$\text{constant} \times \sum_{m=-\infty}^{\infty} \int_{\bar{\tau}_m}^{\bar{\tau}_{m+1}} \mathrm{d}\bar{\tau}\, (-1)^m [\exp(\pi \mathrm{i}(2n+1)\alpha\, \mathrm{e}^{\bar{\tau}}/L) \exp(\mathrm{i}\alpha \Delta E \bar{\tau})]$$

where $\bar{\tau}_m$ is defined as with the untwisted field. We evaluate the integral as before, by splitting the sums into the $m=0$ term and the $|m|>0$ term. The former terms are

$$b^{\mathrm{i}\alpha\Delta E}\, \mathrm{e}^{-\pi\Delta E\alpha/2}[\Gamma(-\mathrm{i}\Delta E\alpha, \mathrm{i}b \exp(-\cosh^{-1}(L/\alpha))) - \Gamma(-\mathrm{i}\Delta E\alpha, \mathrm{i}b \exp(\cosh^{-1}(L/\alpha)))] \qquad (3.4)$$

and its complex conjugate respectively, where we now have $b \equiv \pi(2n+1)\alpha/L$. The $|m|>0$ term is identical to (2.12) with a replaced by b, and a factor $(-1)^m$ in the summand arising from the antiperiodicity of the boundary conditions. Its presence precludes the performance of the summations. We proceed by splitting the sums into even and odd m parts; rearranging we find the sums reduce to

$$\begin{aligned}\mathrm{e}^{-\pi\Delta E\alpha/2}\Bigg(&\sum_{m=1}^{\infty} [\Gamma(-\mathrm{i}\alpha\Delta E, \mathrm{i}bf(m)) - \Gamma(-\mathrm{i}\alpha\Delta E, \mathrm{i}b/f(m)) \\ &- \Gamma(-\mathrm{i}\alpha\Delta E, \mathrm{i}bf(m+1)) + \Gamma(-\mathrm{i}\alpha\Delta E, \mathrm{i}b/f(m+1))] \\ &+ 2\sum_{m=1}^{\infty} [\Gamma(-\mathrm{i}\alpha\Delta E, \mathrm{i}bf(2m)) - \Gamma(-\mathrm{i}\alpha\Delta E, \mathrm{i}b/f(2m)) \\ &- \Gamma(-\mathrm{i}\alpha\Delta E, \mathrm{i}bf(2m-1)) + \Gamma(-\mathrm{i}\alpha\Delta E, \mathrm{i}b/f(2m-1))]\Bigg) b^{\mathrm{i}\Delta E\alpha}\end{aligned} \qquad (3.5)$$

and its complex conjugate. The first sum is identical to the corresponding sum for the untwisted case (2.12), and leads to a term similar to the first term of (2.13) in the response. The total contributions to the response are therefore

$$\begin{aligned}\mathrm{e}^{-\pi\Delta E\alpha/2} b^{\mathrm{i}\Delta E\alpha}\Bigg(&\Gamma(-\mathrm{i}\Delta E\alpha) + 2\sum_{m=1}^{\infty} [\Gamma(-\mathrm{i}\alpha\Delta E, \mathrm{i}bf(2m)) - \Gamma(-\mathrm{i}\Delta E\alpha, \mathrm{i}b/f(2m)) \\ &- \Gamma(-\mathrm{i}\alpha\Delta E, \mathrm{i}bf(2m-1)) + \Gamma(-\mathrm{i}\alpha\Delta E, \mathrm{i}b/f(2m-1))]\Bigg)\end{aligned} \qquad (3.6)$$

and its complex conjugate.

Unfortunately the remaining sum cannot be evaluated exactly. However, for $L \gg \alpha$ an approximate expression can be obtained. In this limit

$$f(m) = \exp[\cosh^{-1}(mL/\alpha)] = (mL/\alpha) + ((mL/\alpha)^2 - 1)^{1/2} \simeq 2mL/\alpha.$$

From the definition of b, this implies

$$ibf(m) \simeq 2\pi i m(2n+1), \qquad ib/f(m) \simeq -2\pi i m(2n+1).$$

Using the analytically continued integral representation for the Γ-function gives as an approximation of the summand in (3.6)

$$\int_0^\infty \mathrm{d}t\, \mathrm{e}^{-t} \Big\{[t+4\pi i m(2n+1)]^{-\mathrm{i}\alpha\Delta E-1} - [t+2\pi\mathrm{i}(2m-1)(2n+1)]^{-\mathrm{i}\alpha\Delta E-1} + [t-2\pi\mathrm{i}(2m-1)(2n+1)]^{-\mathrm{i}\alpha\Delta E-1} - [t-4\pi i m(2n+1)]^{-\mathrm{i}\alpha\Delta E-1}\Big\}\, b^{\mathrm{i}\Delta E\alpha}.$$

The sum in (3.6) may now be evaluated explicitly to yield

$$\begin{aligned}\mathrm{e}^{-\pi\Delta E\alpha/2}\Bigg[&(\pi(2n+1)\alpha/L)^{+\mathrm{i}\alpha\Delta E}\Gamma(-\mathrm{i}\alpha\Delta E)\\ &+2^{-\mathrm{i}\alpha\Delta E}\Bigg((+2\pi\mathrm{i}(2n+1))^{-\mathrm{i}\alpha\Delta E-1}\int_0^\infty \mathrm{d}t\{\zeta(1+\mathrm{i}\alpha\Delta E, +t/4\pi\mathrm{i}(2n+1))\\ &-\zeta(1+\mathrm{i}\alpha\Delta E, [+t+2\pi\mathrm{i}(2n+1)]/4\pi\mathrm{i}(2n+1))\\ &-[+2\pi\mathrm{i}(2n+1)/t]^{1+\mathrm{i}\alpha\Delta E}\}\,\mathrm{e}^{-t}\\ &-(-2\pi\mathrm{i}(2n+1))^{-\mathrm{i}\alpha\Delta E-1}\int_0^\infty \mathrm{d}t\{\zeta(1+\mathrm{i}\alpha\Delta E, -t/4\pi\mathrm{i}(2n+1))\\ &-\zeta(1+\mathrm{i}\alpha\Delta E, [-t+2\pi\mathrm{i}(2n+1)]/4\pi\mathrm{i}(2n+1))\\ &-[-2\pi\mathrm{i}(2n+1)/t]^{1+\mathrm{i}\alpha\Delta E}\}\,\mathrm{e}^{-t}\Bigg)\, b^{+\mathrm{i}\alpha\Delta E}\Bigg]\end{aligned} \tag{3.7}$$

and its complex conjugate, where we have used the identity

$$\sum_{m=0}^{\infty} (-1)^m/(m+a)^s = 2^{-s}[\zeta(s, a/2) - \zeta(s, (a+1)/2)], \qquad \operatorname{Re} s > 0.$$

The integrals in (3.7) are manifestly convergent. We will not evaluate them, but will show that they contribute only to the transient part of the detector's response. Using the definition of b in (3.7), we see that the transition amplitude may be written as

$$\begin{aligned}\mathrm{i}c\alpha\langle E|m(0)|E_0\rangle \exp(-\pi\Delta E\alpha/2)\, b^{\mathrm{i}\Delta E\alpha}[&-\Gamma(-\mathrm{i}\Delta E\alpha)\\ &+[2\pi\mathrm{i}(2n+1)]^{\mathrm{i}\Delta E\alpha+1}2^{-\mathrm{i}\Delta E\alpha}B(\Delta E\alpha, (2n+1))\\ &+[-2\pi\mathrm{i}(2n+1)]^{-(\mathrm{i}\Delta E\alpha+1)}2^{-\mathrm{i}\Delta E\alpha}B(\Delta E\alpha, -(2n+1))]/[2\pi(2n+1)]^{1/2}\end{aligned}$$

where

$$\begin{aligned}B(\Delta E\alpha, (2n+1)) = \int_0^\infty &\mathrm{e}^{-t}[\zeta(1+\mathrm{i}\alpha\Delta E, t/4\pi\mathrm{i}(2n+1))\\ &-\zeta(1+\mathrm{i}\alpha\Delta E, [-t+2\pi\mathrm{i}(2n+1)]/4\pi\mathrm{i}(2n+1))]\,\mathrm{d}t\end{aligned} \tag{3.8}$$

and (minus) its complex conjugate for the $\langle \bar{1}_k|\phi(x)|0\rangle$ term. To evaluate the transition probability of the detector, we use

$$P=\alpha^2c^2|\langle E|m(0)|E_0\rangle|^2\sum_{k=1}^{\infty}\left\{\left|\sum_{m=-\infty}^{\infty}\int_{\bar\tau_m}^{\bar\tau_{m+1}}\mathrm{d}\bar\tau\,\mathrm{e}^{\mathrm{i}\Delta E\alpha\bar\tau}\langle\vec 1_k|\phi[x]|0\rangle\right|^2\right.$$

$$\left.+\left|\sum_{m=-\infty}^{\infty}\int_{\bar\tau_m}^{\bar\tau_{m-1}}\mathrm{d}\bar\tau\,\mathrm{e}^{\mathrm{i}\Delta E\alpha\bar\tau}\langle\overleftarrow{1}_k|\phi[x]|0\rangle\right|^2\right\}$$

which gives

$$P=c^2|\langle E|m(0)|E_0\rangle|^2\left\{[(E-E_0)(\exp(2\pi\alpha(E-E_0))-1)]^{-1}\sum_{n=1}^{\infty}(\alpha/n)+\text{finite terms}\right\}. \tag{3.9}$$

The 'finite terms' are simple combinations from the B-functions in (3.8). Only the first term in (3.9) has the logarithmic divergence characteristic of a constant transition rate. All the other terms in the reponse are finite. So, the detector's response has a thermal time-independent component plus a time-dependent transient centred on $\tau=0$. The twisted field response is composed of the untwisted field response plus a transient 'twist term' localised around $\tau=0$. This might be taken to suggest that the detector could determine the $S^1\times R^1$ topology of spacetime (by detecting the presence of twisted states) from a local experiment. It must be remembered though that the treatment discussed here refers to the detector's response over the entire world-line ($-\infty<\tau<\infty$) during which the detector will encircle the 'cylinder' repeatedly. In that sense it will be aware of the non-trivial topology, even though the 'twist term' is independent of L.

It may be argued, on grounds of physical consistency, that for the twisted field one should use a detector coupling which is 'gauge invariant' in the sense that it remains unchanged as the field twists flip the sign of ϕ with successive circuits around the cylinder. A detector in which the monopole moment $m(\tau)$ couples to ϕ^2 is such a gauge invariant detector. However, it is easy to show that this quadratically coupled detector also responds to the field twists.

In (2.2) we gave an expression for the transient rate per unit detector time at time τ_0. From (3.9) we know that the DeWitt detector's transition rate at time τ_0 has τ_0-dependent and τ_0-independent components. Further we can split away the τ-dependent twist terms of the response. Therefore, for the DeWitt detector in this situation, (2.2) has the general form

$$c^2|\langle E|m(0)|E_0\rangle|^2\int_{-\infty}^{\infty}\mathrm{d}\eta\,\mathrm{e}^{-\mathrm{i}\Delta E\eta}$$

$$\times\{[A(\eta)+C(\eta,\tau_0)]\theta(-\eta)+[A(\eta)+C(-\eta,\tau_0)]\theta(\eta)\} \tag{3.10}$$

where $A(\eta)$ contains the time-independent (thermal) component of the response and $C(\eta,\tau_0)$ the twisted τ_0-dependent component.

For the quadratic detector, after removing the vacuum divergence the transition rate per unit detector time at time τ_0 is (Hinton 1984)

$$2c^2|\langle E|m(0)|E_0\rangle|^2\int_{-\infty}^{\infty}\mathrm{d}\eta\,\mathrm{e}^{-\mathrm{i}\Delta E\eta}$$

$$\times\{[G^+(x(\eta+\tau_0),x(\tau_0))]^2\theta(-\eta)+[G^+(x(\tau_0),x(\tau_0-\eta))]^2\theta(\eta)\} \tag{3.11}$$

which, for the twisted case we are considering, leads to an expression similar to (3.10) but quadratic rather than linear in $(A+C)$. Now, for the quadratic detector to remain unaware of the field twists, it must respond to the twisted and untwisted fields in the same way. This requires, taking an inverse Fourier transform and assuming $C \neq 0$:

$$2A(\eta)+C(\eta, \tau_0)=0 \qquad \forall \tau_0$$

which is obviously not the case.

We conclude that the 'gauge-invariant' quadratic detector can recognise whether or not the field is twisted.

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