either linewidth or redshift with time, because the time-scale of such variations may lead to masses for the QSOs. This reasoning probably cannot be applied to objects for which $z_{em} > z_{abs}$ because the absorbing gas in these QSOs is moving outward, and therefore the likelihood that such gas is moving only under the influence of gravity is much less than in the opposite situation. The fact that Ton 1530 and B 194 have absorption redshifts for which both $z_{abs} > z_{em}$ and $z_{abs} < z_{em}$ indicates that the infall and outflow of material may be a continuing cycle. This suggests an evolution for the gas associated with QSOs in which it is accreted by gravitation, and then expelled by some mechanism such as particle pressure[7].

If the interpretation of the $z_{abs} > z_{em}$ phenomenon is correct, the need to postulate the creation of matter in the centre of QSOs[8] would be alleviated somewhat, because it would attest to both the influx of large quantities of gas toward the central regions and the existence of sufficiently large central masses to satisfy the energy requirements of the QSOs.

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Hoyle–Narlikar Theory of Gravitation

HARRIS[1] has given a treatment of the Bohr quantization of a spherically symmetric mass distribution surrounded by a gravitational field in the Einstein theory. One result is to provide a relationship between cosmological and elementary particle parameters.

This note concerns the Bohr quantization of the gravitational theory of Hoyle and Narlikar[2]. This theory derives from a direct interparticle action

$$S = \sum_{a<b}\sum \iint \tilde{G}(A,B)\mathrm{d}a\,\mathrm{d}b \qquad (1)$$

summed once over all pairs of particles a,b, $\mathrm{d}a\mathrm{d}b$ are world line elements and $\tilde{G}$ is a time-symmetric scalar two-point Green function satisfying the equation

$$g^{ij}\tilde{G}(X,A)_{;ij} + \tfrac{1}{6}R(X)\tilde{G}(X,A) = -(-g)^{-\frac{1}{2}}\delta^4(X,A) \qquad (2)$$

where R is the scalar curvature and $\bar{g}$ the determinant of g^{ij}.

The theory is Machian in the sense that the equations of motion do not exist in the limit of a one particle universe. We cannot therefore consider the single mass case of Harris, but must account for all masses $b \neq a$ in the univ se. The mass of particle a is given by

$$m_a(X) = -\sum_{b\neq a}\int \tilde{G}(X,B)\mathrm{d}b \qquad (3)$$

and is a function of position.

Although the theory is nonlinear, so that the contributions from the right-hand side of equation (3) may not be superposed directly, such a treatment of Machian theories is often given as an approximation and has been discussed in this case by Hawking[3]. Although the advanced contribution to m_a diverges in this approximation, the retarded contribution is finite, and given by

$$m_a = N/H^2 \qquad (4)$$

where H is the Hubble constant and, in the steady-state cosmology, N is the constant particle density

$$N(X) = \sum_a \int \delta^4(X,A)[-g(X,A)]^{-\frac{1}{2}}\,\mathrm{d}a \qquad (5)$$

If we assume that in a rigorous treatment of the Hoyle–Narlikar gravitation theory the advanced contribution causes no embarrassment we may therefore proceed with the fully retarded part (4). Remembering that this is only a linear approximation, we substitute (3) and (4) into the total action (1)

$$S = \frac{N}{H^2}\sum_a \int \mathrm{d}a \qquad (6)$$

If we work with the conformally flat form of the cosmological metric, it is easily shown[3] that the right-hand side of (6) may be written

$$\frac{N^2}{H^2}\int_0^{1/H}\int_{-\infty}^{-1/H}\frac{4\pi r^2\,\mathrm{d}r}{(\tau H)^4}\,\mathrm{d}\tau$$

where (5) has also been used. This is immediately evaluated to give

$$\frac{N^2}{3H^3}\cdot\frac{4\pi}{3}\left(\frac{1}{H}\right) = \frac{N_0^2}{4\pi}$$

where N_0 is the total number of particles in the universe. The net action for each particle is then Bohr quantized

$$N_0/4\pi = nh \qquad (7)$$

Substituting this into (4) for the mass

$$m_a = 3nhH \qquad (8)$$

We now have to decide on the value of n. If the theory is Machian we expect $m_a \to 0$ as $N_0 \to 0$ so that n should be a positive power of N_0. We make the *ad hoc* assumption that n is an integer multiple of $\sqrt{N_0}$. This quantity is the mean fluctuation in the total number of particles, N_0, and frequently appears in this type of analysis, although as yet its significance is not understood.

Then

$$m_a = 3n_0\sqrt{N_0}\,hH = 3n_0\sqrt{N_0}\,hH/c^2$$

in c.g.s. units, where n_0 is a smaller integer.

Numerically this gives

$$m_a \sim 10^{-24}\ \mathrm{g}$$

or about the mass of elementary particles. This result becomes still more striking if we use the well known "coincidence"

$$\sqrt{N_0} \sim \frac{c/H}{4\pi e^2/m_e c^2}$$

(m_e = mass of electron). Then

$$m_a \sim \tfrac{1}{2}\frac{\hbar c}{e^2}\,n_0\,m_e$$

$$\sim \tfrac{1}{2}\times 137\,n_0\,m_e$$

which is just Nambu's formula[4], originally derived on empirical grounds to describe the mass spectrum of elementary particles.

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Energy Levels and Lorentz Invariance

AN atom is in an excited state S′ with energy E'. It falls to a state S with lower energy E and emits a photon with frequency ν. According to a basic postulate of quantum mechanics, this frequency is related to the energies by

$$h\nu = E' - E \qquad (1)$$

The theoretical calculation of E and E' is carried out in the rest-frame of the atom, and so it would appear that ν in equation (1) is the frequency observed in that frame. Conservation of momentum, however, demands that the atom recoils when the photon is emitted, and so there is not one rest-frame but two—the rest-frame of S′ and the rest-frame of S. It is true that the recoil is small and consequently the two rest-frames are nearly the same. But in spectroscopy wavelengths are sometimes given with 8-figure accuracy, and it is interesting to inquire whether such (or higher) accuracy is consistent with neglect of the small difference between the two rest-frames.

To discuss the recoil relativistically we must speak of 4-momentum, and it is convenient to take units for which $c=1$ and to use imaginary time ($x_4 = it$). Latin suffixes range 1, 2, 3, 4. Let p'_r, p_r be the 4-momenta of the atom in the states S′, S, and let k_r be the 4-momentum of the emitted photon. Then

$$p'_r p'_r = -m'^2,\ p_r p_r = -m^2,\ k_r k_r = 0 \qquad (2)$$

where m', m are the proper masses of S′, S. (Because $c=1$, proper mass and proper energy are merely different names for the same thing.) Conservation of momentum and energy demands that

$$p'_r = p_r + k_r \qquad (3)$$

or equivalently

$$p_r = p'_r - k_r \qquad (4)$$

Squaring this and using equation (2), we get

$$m^2 = m'^2 + 2p'_r k_r \qquad (5)$$

The frequency ν of the emitted photon is given by $ih\nu = k_4$, the fourth component of a 4-vector, and, if we are to make a significant spectroscopic prediction about this frequency, we must make up our minds what frame of reference to use. If we were concerned with the excitation of an atom (S→S′), it would be natural to use the rest-frame of S. But we are concerned with emission, and there can be no doubt that the rest-frame of S′ is the one to use—it is the mass-centre frame for the event. In this frame we have

$$p'_r = (0,\ 0,\ 0,\ im'),\ p'_r k_r = -m'h\nu \qquad (6)$$

and so equation (5) gives

$$h\nu = \tfrac{1}{2}(m'^2 - m^2)/m' \qquad (7)$$

If we knew the proper masses, m', m, we could calculate (without reference to equation (1)) the frequency ν, measured in the rest-frame of the atom before emission.

We have now before us two formulae for the frequency of the emitted photon, the formula (1) which will be found in any textbook on quantum mechanics[1] and the formula (7) obtained as above by a simple application of the conservation of 4-momentum, the only quantum feature being the assumption that the energy of a photon is $h\nu$. They cannot both be right, unless the two right-hand sides happened to be equal. That can hardly be the case, but they are nearly equal in practice. This near-equality has been fortunate, for it has permitted quantum theory to grow from strength to strength on the basis of a formula which, to a relativist, cannot but appear rather grotesque. I propose to compare the two formulae, that is, the formulas (7) and (1), rewritten as

$$h\nu_0 = E' - E \qquad (8)$$

to avoid using the same symbol for different things. I shall call ν the true frequency and ν_0 the conventional frequency; E' and E are energy levels of the atom, calculated by quantum theory.

No comparison can be made until we answer the following basic question: What is the proper mass (equivalently, rest-mass or rest-energy) of an atom in a state S with energy E ?

It appears that if $m_1, m_2, \ldots m_n$ are the proper masses of the particles of which the atom is composed, then the proper mass m of the atom is

$$m = M + W \qquad (9)$$

where

$$M = m_1 + m_2 + m \ \ldots\ + m_n \qquad (10)$$

and W is the (negative) binding energy or mass defect[2]. But equation (9) as it stands does not mean much. It might be taken as the definition of W, and the fact that W is negative merely implies that the mass of the atom is less than the sum of the masses of its constituent particles. In fact, equation (9) becomes significant only when we identify W with the energy E as in equation (8) and write for the two proper masses in equation (7).

$$m' = M + E',\ m = M + E \qquad (11)$$

I have searched without success for an authoritative statement that $W=E$. Perhaps it is accepted without question (see ref. 3). These authors use equation (8) above without mention of the two frames of reference, but the difference between them may be covered by the approximation used.

Substitution from equation (11) in equation (7) gives

$$h\nu = (E'-E)[M + \tfrac{1}{2}(E+E')](M+E')^{-1} \qquad (12)$$

and so by equation (8) the ratio of the true frequency to the conventional frequency is given accurately by

$$\nu/\nu_0 = [1 + \tfrac{1}{2}(E+E')/M](1+E'/M)^{-1} \qquad (13)$$

Now E/M and E'/M are in fact dimensionless small quantities of the same order of magnitude, and expansion of equation (13) gives

$$(\nu_0 - \nu)/\nu_0 = \tfrac{1}{2}(E'-E)/M + O[(E/M)^2] \qquad (14)$$

or, with equation (8)

$$(\nu_0 - \nu)/\nu_0 = \tfrac{1}{2}h\nu_0/M + O[(h\nu_0/M)^2] \qquad (15)$$

This may be called the fractional error arising from the use of the conventional frequency instead of the true frequency.

To appreciate the magnitudes involved, let us restore c and use c.g.s. units, so that

$$(\nu_0 - \nu)/\nu_0 = K + O(K^2) \qquad (16)$$

where

$$K = \frac{h\nu_0}{2Mc^2} = \frac{h}{2cM\lambda} \qquad (17)$$