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LETTER TO THE EDITOR

Journey through a black hole

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Abstract. The conjecture that an object may pass through a black hole and enter 'another universe' is re-examined by computing the quantum stress tensor for an electrically charged black hole in thermodynamic equilibrium with a de Sitter horizon. The conjecture is confirmed if a modification is made to the trace anomaly, as suggested by a Euclidean treatment.

There has long been speculation that black holes which rotate and/or carry electric charge can act as 'wormholes' or 'space bridges' to other universes. This speculation is based on certain exact solutions which possess analytic extensions that join our spacetime region to other asymptotically flat regions through the interior of the hole. A more careful study, however, casts doubt on whether a black hole could ever be a wormhole in practice. The problem arises because of the existence in these models of an inner, or Cauchy, horizon. This acts as a surface of infinite blueshift, causing any radiation or matter that falls into the hole from the surrounding universe to pile up along the horizon without limit, presumably either producing a singularity, or modifying the internal structure of the hole by back reaction, in such a way that the wormhole could not actually be navigated by a particle or observer.

The speculation was originally formulated and investigated in the context of classical black holes [1, 2]. Following Hawking's discovery of black-hole radiance [3], however, a quantum treatment became necessary. Birrell and Davies [4] evaluated the expectation value of the stress-energy-momentum tensor $T^{\mu\nu}$ of a scalar quantum field in the gravitational background of an electrically charged, non-rotating hole. In order for the black hole to avoid evaporation, the quantum field must be in the so called Hartle-Hawking state [5], corresponding to the hole being in thermodynamic equilibrium with a surrounding heat path. It is readily shown [4] that the ingoing energy flux, T_{vv}, for this state vanishes at the surface of the hole (i.e. the event horizon): there is no net flux of energy into the hole from the surrounding universe. Nevertheless, Birrell and Davies were able to show that even in this case, T_{vv} still blows up on the inner horizon. The reason can be traced to the existence of quantum vacuum stress induced by the spacetime curvature of the hole, which is non-vanishing (indeed, divergent on the inner horizon) even when there is no net energy flux. The conclusion was that there is still less reason to believe the space bridge conjecture using a quantum treatment than there was from the classical studies.

However, all previous work on this topic made the assumption that spacetime was asymptotically flat. Recent work by one of us [6] suggests that if this assumption is relaxed, then there is a loophole in the argument that travel through a black hole into another universe is prevented by energy pile-up on the inner horizon. Mellor and Moss [7, 8] have analysed a class of exact solutions corresponding to black holes in de Sitter-like universes. These models possess three horizons: a cosmological horizon, an outer black-hole horizon and an inner (Cauchy) horizon. The surface gravities of the cosmological and outer black-hole horizons are equal, corresponding to a state of thermodynamic equilibrium between the black hole and the cosmological horizon. Using a classical analysis, it can be shown [6] that this matching of surface gravities suppresses the divergence of the field energy along the Cauchy horizon, opening up the prospect that the Mellor–Moss solutions might provide genuine 'space bridges' to other universes. The purpose of this letter is to re-examine this scenario using a quantum treatment.

We shall consider the spherically symmetric static Mellor–Moss solution corresponding to a black hole of charge Q and mass M:

$$ds^2=(1-2M/r+Q^2/r^2-\tfrac{1}{3}\Lambda r^2)\,dt^2-(1-2M/r+Q^2/r^2-\tfrac{1}{3}\Lambda r^2)^{-1}\,dr^2 - r^2(d\theta^2+\sin^2\theta\,d\phi^2) \quad (1)$$

$$M^2=Q^2 \quad (2)$$

$$\kappa_1=\kappa_2=\{\tfrac{1}{3}[1-4M(\tfrac{1}{3}\Lambda)^{1/2}]\}^{1/2}. \quad (3)$$

The cosmological and outer black-hole horizons are located at r_1 and r_2 respectively, and their corresponding surface gravities are denoted by κ_1 and κ_2. The region between the horizons is filled with thermal radiation at the common Hawking temperature $\kappa/2\pi$. A Penrose diagram for this spacetime is shown in figure 1.

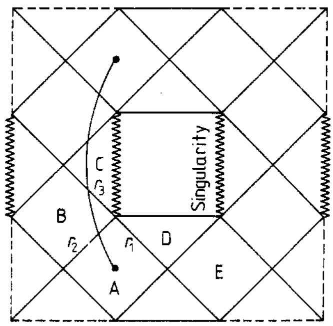

Figure 1. Part of the Penrose diagram for the black-hole–de Sitter spacetime. Regions A, D and E lie outside the holes whilst regions B and C lie inside.

In the two-dimensional spacetime obtained by suppressing the angular coordinates, the stress–energy–momentum tensor for a conformally coupled massless scalar field can be obtained exactly. Although this is a restricted model, it is widely supposed to reflect the essential features of the full four-dimensional case.

Subject to time invariance the conservation condition

$$T^{\mu\nu}{}_{;\nu}=0 \quad (4)$$

can be integrated to give [9]:

$$T^r{}_t=\text{constant} \quad (5)$$

$$g_{tt}T^r_r=\tfrac{1}{2}\int g_{tt,r}T\,dr+\text{constant} \quad (6)$$

where T is the trace T^{μ}_{μ}. Using the relation

$$T_{vv}=\tfrac{1}{4}g_{tt}T+\tfrac{1}{2}T^r_t-\tfrac{1}{2}g_{tt}T^r_\tau \tag{7}$$

one obtains from (5)-(7)

$$T_{vv}(r)=\tfrac{1}{4}g_{tt}T-\tfrac{1}{4}\int g_{tt,r}T\,\mathrm{d}r+c \tag{8}$$

where c is a constant. The trace is determined entirely by the conformal anomaly:

$$T=-R/24\pi=-g_{tt,rr}/24\pi. \tag{9}$$

Thus (8) gives

$$T_{vv}=(\kappa^2(r)-\tfrac{1}{2}g_{tt}g_{tt,rr})/48\pi+c \tag{10}$$

using the definition of the surface gravity $\kappa=\tfrac{1}{2}|g_{tt,r}|$.

Interest centres on the behaviour of T_{vv} in the vicinity of the horizons. At the cosmological and Cauchy horizons, r_1 and r_3 respectively, $v\to\infty$. The locally measured stress seen by a freely falling observer crossing this horizon will therefore diverge unless $T_{vv}=0$ [9]. From (10) it follows that

$$T_{vv}(r_1)=\kappa_1^2/48\pi+c \tag{11}$$

$$T_{vv}(r_3)=\kappa_3^2/48\pi+c \tag{12}$$

where κ_3 is the surface gravity on the Cauchy horizon. It is clear from (11) and (12) that we may choose the constant c so that T_{vv} vanishes either on r_1 or on r_3, but not on both, unless $\kappa_1=\kappa_2=\kappa_3$. A straightforward calculation using the solution (1)-(3) gives

$$\kappa_3=\{\tfrac{1}{3}\Lambda[1+4M(\tfrac{1}{3}\Lambda)^{1/2}]\}^{1/2}. \tag{13}$$

A comparison of (3) and (13) shows that $\kappa_1=\kappa_2=\kappa_3$ only if $\Lambda=0$, in which case the condition (2) corresponds to a naked singularity, i.e. the horizons disappear altogether. The conclusion would therefore seem to be that unavoidable quantum vacuum effects cause divergent terms in the locally measured stress tensor at the Cauchy horizon, and thus prevent an observer from journeying through the Cauchy horizon to 'another universe'. The general impossibility of constructing a quantum state that gives a finite stress tensor on all three horizons of black-hole–de Sitter spaces has been pointed out by Hiscock [10].

It is curious that the quantum and classical treatments give differing results on this system. One might, however, question whether the quantum treatment given here is completely satisfactory. Could it be that a different choice of quantum state, or the use of a Euclidian approach, might give a result consistent with the classical theory? One way in which a difference might occur concerns the anomalous trace, the form of which is crucial in determining the stress tensor. As seen from (12), the trace is proportional to the curvature scalar, R. The Euclidian section of the portion of spacetime between the inner and outer black hole horizons is shown in figure 2. A distinctive feature of this manifold is the presence of conical singularities at the points corresponding to the two horizons. These singularities will exist so long as $\kappa_2\neq\kappa_3$. They arise for the following reason. The time coordinate t becomes an angular coordinate τ in the Euclidian section. Hence it is necessary to make an identification of the form $\tau\to\tau+2\pi/\alpha$. The periodicity parameter α may be chosen to correspond

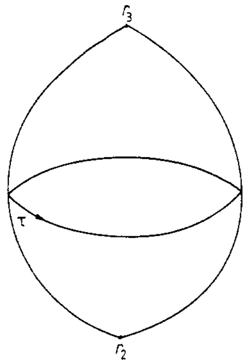

Figure 2. The Euclidian section of the interior region.

to the temperature (i.e. surface gravity) on the outer horizon r_2, in which case the singularity associated with the outer horizon is removed. Alternatively α may be chosen to correspond to κ_3, in which case the singularity corresponding to the Cauchy horizon is removed. So long as $\kappa_2 \neq \kappa_3$ it is not possible to choose a value for α which removes both singularities. For a general value of α there will be two such singularities.

The presence of conical singularities in the Euclidian manifold introduces δ functions into its curvature scalar R. If these δ-function terms are retained when analytically continuing back to the Lorentzian section, they will introduce similar δ-function terms into the anomalous trace at the horizons. Because of the ambiguity in the choice of α there will be a corresponding ambiguity in the coefficients of these terms. Interestingly, it is possible to choose α so as to make T_{vv} regular on both horizons, apart from δ-function singularities. Such singularities are not too serious because they imply finite values for locally measured physical quantities when integrated across the horizons.

To see how this works, note that the Euclidean metric in the region between r_2 and r_3 corresponding to solution (1)-(3) is

$$ds^2 = \kappa_2^{-2}(d\rho^2 + \kappa_2^2\rho^2\, d\tau^2) \tag{14}$$

from which we obtain the singular term in R of

$$4(\alpha - \kappa_2)\kappa_2\delta(\rho^2). \tag{15}$$

Choosing α to be $(\kappa_2^2 + \kappa_3^2)/2\kappa_2$, and analytically continuing back to the Lorentz section, this becomes

$$\kappa_2^{-1}(\kappa_3^2 - \kappa_2^2)\delta(r - r_2). \tag{16}$$

If, in (8) and (9), R is augmented by the term (16), we obtain in place of (11) and (12)

$$T_{vv}(r_1) = T_{vv}(r_3) = \kappa_3^2/24\pi + c \tag{17}$$

so by choosing $c = -\kappa_3^2/24\pi$ we can arrange that T_{vv} vanishes on both horizons.

The conclusion would seem to be that, on the basis of a Euclidian calculation, there exists a quantum state for which the stress tensor can (to within δ-function terms) be finite on all three horizons, thus in principle permitting passage from one 'universe' to another. We note the curious feature that T_{vv} jumps discontinuously across the outer black-hole horizon. This implies that an observer could determine, by studying the quantum stress tensor, the location of the event horizon. But an event horizon is supposedly a global feature, with no local physical significance. The local significance is acquired here because, in carrying out the Euclidian transformation, the horizon is mapped to a single point. Thus in the Euclidian section, the horizon is indeed a local feature.

References

[1] Simpson M and Penrose R 1973 *Int. J. Theor. Phys.* **1** 183
[2] Chandrasekhar S and Hartle J B 1982 *Proc. R. Soc.* A **384** 301
[3] Hawking S W 1975 *Commun. Math. Phys.* **43** 199
[4] Birrell N D and Davies P C W 1978 *Nature* **272** 35
[5] Hartle J B and Hawking S W 1976 *Phys. Rev.* D **13** 2188
[6] Moss I G 1989 Journeys beyond the Cauchy horizon *University of Newcastle Preprint*
[7] Mellor F and Moss I G *Phys. Lett.* B in press
[8] Mellor F and Moss I G 1989 *Class. Quantum Grav.* to be published
[9] Davies P C W, Ford L-H and Page D N 1986 *Phys. Rev.* D **34** 1700
[10] Hiscock W A 1989 Quantum instability of gravitational collapse in de Sitter space *Montana State University Preprint*