Proc. R. Soc. Lond. A **377**, 147–149 (1981)
Printed in Great Britain

Why is the cosmological constant so small?

BY P. C. W. DAVIES AND S. D. UNWIN
School of Physics, The University, Newcastle upon Tyne NE1 7RU, U.K.

(*Communicated by S. K. Runcorn, F.R.S. – Received* 17 *February* 1981)

Quantum field effects inevitably contribute to the cosmological constant Λ, but typical contributions are some fifty orders of magnitude greater than the observed upper limit on Λ. This paradox is re-examined in the context of twisted field configurations.

Many authors have pointed out that modern gauge theories of fundamental forces involving Higgs scalar fields and spontaneously broken symmetry lead to terms in $T_{\mu\nu}$ that may be identified with Einstein's cosmological constant Λ (Kirzhnits & Linde 1976; Canuto & Lee 1977; Coleman & De Luccia 1980). For example, in the simple σ-model, with a single self-interacting, real scalar field ϕ, the effective potential $V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4$ produces a non-zero vacuum expectation value $\sigma = \langle 0|\phi|0\rangle$, when $\mu^2 > 0$. At high temperatures, this broken symmetry will be restored (Kirzhnits & Linde 1976).

The cosmological constant is related simply to σ:

$$\Lambda = 8\pi G \tfrac{1}{4}\lambda(\sigma^2 - \mu^2/\lambda)^2 + \epsilon_0 \tag{1}$$

(we use $\hbar = c = 1$), where ϵ_0 is an adjustable constant representing contributions to Λ from vacuum energy renormalization. Its value must be fixed by observation.

The μ^4-term in (1) will contribute to Λ typically about $10^{-6}\,\mathrm{cm}^{-2}$ (Canuto & Lee 1977), whereas the observed value is less than about $10^{-57}\,\mathrm{cm}^{-2}$. These facts can be reconciled by 'fine-tuning' ϵ_0 to almost exactly cancel this huge quantity. However, σ is not fixed, but temperature-dependent. If ϵ_0 is adjusted so that $\Lambda \approx 0$ at high temperature, when the symmetry is restored and $\sigma = 0$ (a state thought to have prevailed in the early universe), then $\Lambda \approx 10^{-6}\,\mathrm{cm}^{-2}$ today. Conversely, if ϵ_0 is chosen consistent with the currently observed value of $\Lambda \approx 0$, then Λ would have been enormous in the primeval universe.

Why should ϵ_0 have been chosen by nature to cancel terms of order μ^4 so precisely in the low temperature régime? As emphasized by Coleman & De Luccia (1980), this is surely one of the great mysteries about gravity.

In the absence of a fundamental physical reason why Λ should be so small, a possible anthropic explanation suggests itself. Perhaps the excessive smallness of Λ is a feature that only characterizes our particular region of the universe. In other regions this fine-tuning fails and Λ assumes much greater values. But in such regions Λ would dominate the gravitational dynamics, leading to exponential expansion, or (for negative values) collapse into anti-de Sitter space. Probably, values that differ by more than a few orders of magnitude from the observed upper limit in our region would be sufficient to prevent the formation of galaxies, and hence organic life. It is therefore no surprise that we find ourselves in an atypical region of the universe in which fine-tuning of ϵ_0 has produced a negligible Λ.

What evidence is there that Λ might differ substantially from its 'locally' measured value in remote regions of space–time? At first it might appear that departures from homogeneity would result in significant variations in σ throughout space. The scalar wave equation requires, classically,

$$\Box\sigma - \mu^2\sigma + \lambda\sigma^3 = 0, \tag{2}$$

which in Minkowski space possesses the symmetry-broken solution $\sigma = \mu/\lambda^{\frac{1}{2}}$. In curved space–time there will be contributions from $\Box\sigma$, and σ can no longer be constant. However, typical variations, $\Delta\sigma/\sigma$, will be of order $H/\mu \approx 10^{-40}$, where H is Hubble's constant. This leads, via (1), to variations in Λ of about one part in 10^{160} over cosmologically significant length scales. The effect of space–time curvature is clearly negligible except perhaps in the very early stages of the big bang, or near small black holes.

On the other hand, irrespective of the space–time geometry, it is necessary also to take into account the topology of the universe. If the topology of space is nontrivial, then the possibility exists that the universe may admit a twisted field structure (Isham 1978). Since the possible real scalar field configurations in a space–time of underlying manifold $\mathbb{R}^1$ (time) $\otimes$ M (space) are in one-to-one correspondence with the elements of the cohomology group $H^1(M, \mathbb{Z}_2)$, the requirement for the existence of nontrivial scalar field structure in the universe is that the relevant group contains more than one element.

Spontaneous symmetry-breaking involving twisted scalar fields has been considered by Avis & Isham (1978) and Banach (1981). The characteristic feature is that σ *cannot* be spatially constant in the broken symmetry state as, considering this twisted field as the cross-section of a nontrivial real line bundle, it must pass through the null-vector in at least one fibre (Milnor & Stasheff 1974).

For example, in a simplified $S^1 \otimes R^1$ model, σ is described by a periodic elliptic function, which passes through one cycle of oscillation (one 'twist') around the S^1 spatial sections (Avis & Isham 1978). One expects, as a general feature, that broken-symmetry twisted fields would display periodic behaviour over cosmological dimensions.

The amplitude of these variations in σ is of order μ^2/λ. Hence the fine-tuning of ϵ_0 to cancel terms of this order in (1) cannot be effected globally. As a result, Λ will display huge spatial variations. Typically its value will be of order $10^{-6}\,\mathrm{cm}^{-2}$, but in certain 'small' regions where the variation of σ results in almost exact cancellation against ϵ_0, Λ will be close enough to zero for galaxies – and life – to form.

Presumably the size of this equable cosmic oasis needs to be comparable with the Hubble radius, as cosmological surveys do not reveal any systematic gravitational disruption. This implies that the 'radius' of the closed spatial sections must be many orders of magnitude greater than the Hubble radius.

References

Avis, S. J. & Isham, C. J. 1978 *Proc. R. Soc. Lond.* A **363**, 581.
Banach, R. 1981 *J. Phys.* A **14**, 901.
Canuto, V. & Lee, J. F. 1977 *Phys. Lett.* B **72**, 281.
Coleman, S. & De Luccia, F. 1980 *Phys. Rev.* D **21**, 3305.
Isham, C. J. 1978 *Proc. R. Soc. Lond.* A **362**, 383.
Kirzhnits, D. A. & Linde, A. D. 1976 *Ann. Phys.* **101**, 195.
Milnor, J. W. & Stasheff, J. D. 1974 *Characteristic classes*. Princeton University Press.