The arrow of time

Why does time apparently fly one way, when the laws of physics are actually time-symmetrical? Paul Davies proposes a quantum solution, in the Whitrow Lecture 2004, given on 8 October 2004.

ABSTRACT

In daily life the world shows a marked distinction between past and future, but (with a minor exception) the laws of physics are symmetric in time. Gerald Whitrow was among the first to realize that the resolution of this paradox lay with cosmology and the initial conditions at the birth of the universe. But this simple conclusion conceals some deep subtleties that took a new twist with the emergence of quantum cosmology and the inflationary universe scenario. The issue has still not been fully resolved, and remains one of the great outstanding scientific mysteries.

I last saw Gerald Whitrow when I attended an RAS dinner as his personal guest more than 20 years ago. He was a frequent participant in the seminar series I used to run in the Mathematics Department at King's College London, and we would often exchange ideas about the nature of time. I was much influenced by Whitrow's elegant books *The Natural Philosophy of Time* (Whitrow 1961) and *What Is Time?* (Whitrow 1972), and I drew upon his ideas in writing my own volume *The Physics of Time Asymmetry* (Davies 1974). I should mention that I became captivated by the problem of time's arrow in 1968, after hearing Fred Hoyle lecture on the subject at the Royal Society, and it formed the basis of my PhD work.

The recognition that time's arrow does indeed pose a problem dates at least from the year 1854, when Hermann von Helmholtz made what is probably the gloomiest prediction in the history of science. The universe, claimed Helmholtz, is dying. The basis for this apocalyptic pronouncement was the then-new second law of thermodynamics, according to which the entropy of a closed system can never decrease. Entropy is, roughly speaking, a measure of disorder, and when the second law is applied to the universe as a whole it predicts that the universe is steadily degenerating, trapped on a one-way slide towards a state of maximum entropy known as thermodynamic equilibrium. Once this state is achieved the universe will be unable to extricate itself. Hence the final state of equilibrium was dubbed "the heat death" of the universe. The unidirectional transition from order to disorder, culminating in the cosmic heat death, imposes on the universe a pervasive "arrow of time", conventionally taken to point from past to future – in the direction of degeneration. The problem, as it appeared to physicists of the day, and most famously to Ludwig Boltzmann, is that the underlying laws of physics (then assumed to be Newton's laws of mechanics) are symmetric in time. How then can a directed arrow of time emerge from laws that make no distinction between past and future?

Whitrow identified the essence of the solution on pages 160 and 162 of *What Is Time?*, when he clearly stated the problem of time's arrow as one of cosmology: "What the many attempts to analyse the nature of time have shown is that ultimately time must be regarded cosmologically. In the final count, time is a fundamental property of the relationship between the universe and the observer … the ultimate explanation of time's arrow will be found in cosmology."

Whitrow's arrows of time

To understand the cosmological basis of the problem, it is helpful to adopt Whitrow's classification of three different arrows. He distinguished between a historical arrow, a cosmological arrow and a thermodynamic arrow. Let me illustrate each in turn. The historical arrow describes the accumulation of information, or records, over time. A conspicuous example is the cratering of the Moon, which preserves a record of its bombardment by asteroids, comets and meteorites. A less conspicuous, but literally vital, example is the sequence of nucleotides in the genomes of living organisms – a record of evolutionary contingency over billions of years. The accumulation of information in this manner defines an arrow of time that, superficially, might appear to point the opposite way to the thermodynamic arrow. That is, information is the opposite of entropy, so the rise of information looks as if it is "anti-thermodynamic". This is not so: the process of fixing information in a record is itself an irreversible process that generates entropy, so the entropy of the universe as a whole still rises.

The cosmological arrow is easily described: the universe is expanding. Hence the arrow points in the direction of a bigger universe. Some cosmologists have flirted with the idea that if the universe were to one day contract, thereby reversing the cosmological arrow, then the other arrows would reverse too. I believe this view is mistaken. (A spirited recent defence of the reversal hypothesis is given by Price, 1996.)

The thermodynamic arrow has very many manifestations, in both daily life and in astronomy. A simple example is to consider what happens to an egg when dropped on the floor. The egg is easy to beak, but virtually impossible to reassemble. A movie of the egg-breaking episode would, if played in reverse, depict an incredible sequence of events. This applies to most scenes in everyday life: people laugh at movies played backwards because they look so preposterous. The most dramatic irreversible process in our astronomical neighbourhood is the emission of heat and light by the Sun. Every second the Sun radiates 4×10^{26} joules, most of which vanishes into the depths of space, never to return, representing a huge increase in entropy. This profligate expenditure is paid for by the nuclear fuel in the solar core that is used up and its free energy is dissipated around the universe. Over billions of years the Sun steadily and irreversibly burns through its finite stock of fuel, and in time it will burn out and die. The same general story applies to stars across the universe. A significant part of the approach to Helmholtz's cosmic heat death is represented by the ageing and death of stars.

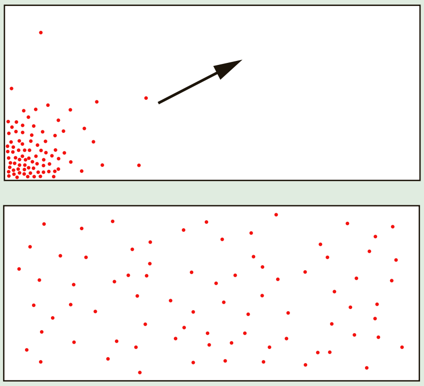
The problem of reversibility

The stark contradiction between the arrow of time and the laws of physics is illustrated most forcefully in the set-up shown in figure 1, where a gas is confined to one corner of a perfectly rigid box and released. It rapidly diffuses throughout the whole volume of the box and eventually settles down to an equilibrium state of uniform density and pressure. This transition provides a clear example of the second law of thermodynamics, because the gas proceeds from a low-entropy, relatively ordered state, to a high-entropy, disordered, equilibrium state. The reverse process – a uniform box of gas in which all the molecules

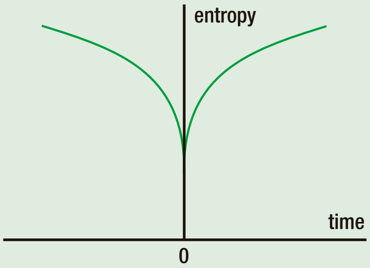
The most dramatic irreversible process in our astronomical neighbourhood is the emission of heat and light by the Sun

The Whitrow Lectures

The Whitrow Lectures were established in memory of Gerald Whitrow, after his death in 2000. Throughout a successful academic career involving astronomy, cosmology, mathematics and the history of science, Whitrow took an abiding interest in the nature of time. The Royal Astronomical Society invites distinguished cosmologists to give the Lectures in fields where Whitrow's ideas and interests have been influential.



1: A gas is isolated in a rigid box. Initially the molecules are confined to one corner of the box (top picture), but as a result of the inter-molecular collisions, the gas rapidly expands into the available space, eventually filling it uniformly (bottom picture). The reverse of this process (bottom picture followed by top picture) would be considered preposterous.



2: At time $t = 0$ a gas is in a low-entropy state. Boltzmann proved that random molecular agitation would (with overwhelming probability) drive the gas to a higher entropy state at $t > 0$. However, because the underlying dynamical laws are symmetric in time, the same reasoning leads to the conclusion that the gas must also have been at a higher entropy state at $t < 0$. This suggests that the arrow of time does not reside with the molecular dynamics, but concerns the question of how the low-entropy state at $t = 0$ was achieved in practice.

suddenly and spontaneously rush into one corner – is never encountered. When we enquire how the gas manages to redistribute itself in the box, we find that this result is brought about by inter-molecular collisions, which transfer energy chaotically among the molecular population, until it is shared democratically. In a famous paper, Boltzmann (1872) proved that the inter-molecular collisions would bring about a rise in entropy, by applying Newtonian mechanics to the collisions and using statistical averaging over a large number of molecules.

No sooner was the ink dry on Boltzmann's paper, however, than a paradox manifested itself. Each individual molecule–molecule collision is *reversible*. If, by some magic, one could reflect a pair of colliding molecules, they would travel back along the same paths to their initial configuration. In principle, the entire population of molecules could be simultaneously rebounded and sent back to the corner of the box. Although technically implausible, there is nothing in the laws of physics to rule it out. (Today, we would use the laws of quantum mechanics rather than Newtonian mechanics to describe these processes, but the latter share with the former the property of time symmetry.) Thus, for every set of motions that raises the entropy, there is an opposite set – perfectly consistent with the laws of dynamics – that lowers it, in violation of the second law of thermodynamics. In fact, by simply reversing the sign of the time parameter in Boltzmann's proof, one may conclude that, just as the entropy will rise from a specified low-entropy state as a result of the inter-molecular collisions, so too must it have *fallen*, from a higher-entropy state, prior to the specified state (see figure 2).

The resolution of the foregoing paradox is to recognize that there is nothing intrinsic to the box of gas that bestows upon it an arrow of time. This is most graphically demonstrated by the work of Poincaré (1893) on this problem, who showed that, given long enough, the gas *would* revisit its initial squashed-in-the-corner state. One way to believe this is to accept that the random agitation of the molecules will always create small fluctuations about uniformity, i.e. little excursions in entropy that go "the wrong way" from the point of view of the second law of thermodynamics. In the case of Brownian motion, for example, a tiny particle suspended in a gas follows a zig-zag path because the bombardment of its surfaces is slightly uneven. Purely on statistical grounds, molecules will sometimes gang up and cooperate in small numbers, slightly lowering the entropy. The rarity of these random fluctuations will rise sharply with the numbers involved, and it is obvious that one would have a long wait before all the molecules in a room rushed to one corner at the same moment purely by chance. A rough measure of the duration between such "recurrences" is 10^N, where N is the number of molecules in the system. For a laboratory box of gas, N might be 10^{23}, so the duration is an exponential of an exponential – a stupendously large number. So large, in fact, that we needn't worry too much about the units of time used to measure it!

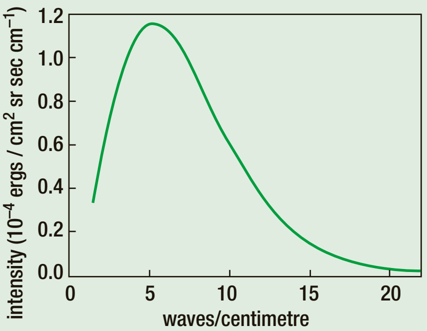
Cosmology to the rescue

The explanation for the arrow of time lies, it would appear, not in the intrinsic dynamics of the gas (or any other system), but in its *special initial conditions*. Clearly it is the case that *if* the gas starts out in a low-entropy state, it will, with overwhelming probability, proceed to a higher entropy state. But we may still enquire as to how the gas achieved its low-entropy state in the first place. One may trace the circumstances of any given system (box of gas, the Sun, a star, etc) to its immediate environment, but ultimately, as Whitrow pointed out, this is a problem of cosmology, because the final environment is the universe as a whole. Boltzmann already confronted the cosmological dimension of the problem in the 19th century, by addressing the issue of how it attained its less-than-maximum entropy state that we see today. His answer (Boltzmann 1897) was to invoke the granddaddy of all fluctuations, a cosmic-wide random excursion from thermodynamic equilibrium, an almost unimaginably rare situation requiring a wait of at least 10 to the power 10^{80} years – or one followed by 10^{80} zeros. There are many reasons why Boltzmann's conjecture won't work, not the least of which is that the universe hasn't been around for the requisite 10 to the power 10^{80} years. Rather, it began with a Big Bang a mere 1.37×10^{10} years ago.

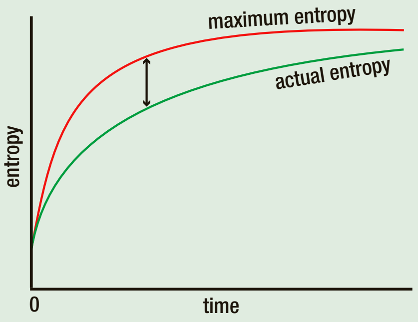
For the correct resolution, we may again turn to Gerald Whitrow. On page 160 of *What Is Time?* he writes: "If there is some deep connection between time and the universe, this may be because time's arrow is associated in some way with the 'initial conditions' that determined the particular universe that actually is, as distinct from any other universe that might have existed in accordance with the same physical principles."

In other words, the universe started out in a special low-entropy state at the Big Bang and has been unwinding ever since, like a gigantic clock running down. So the explanation for the arrow of time rests with the manner of the cosmic birth.

Although this resolution of the reversibility paradox seems plausible enough, an initial consideration of its observational consequences looks problematic. It predicts that the early universe should have been in a much lower entropy state than the universe today. The best information we have about the early universe comes



3: COBE (Cosmic Background Explorer satellite), a precursor of WMAP, measured the spectrum of the cosmic microwave background radiation – the afterglow of the Big Bang – and obtained this textbook curve, corresponding precisely to the spectrum of black-body radiation, a telltale signature of thermodynamic equilibrium. (Courtesy NASA)



4: The entropy of the universe (green curve) rises with time, in accordance with the second law of thermodynamics. However, the maximum possible entropy (red curve) initially rises even faster, as a result of the expansion of the universe. At early times, the matter content (but not the gravitational field) of the universe was close to thermodynamic equilibrium; that is, the actually and maximum possible entropy coincided. Today, there is an entropy gap generated by past cosmological expansion, making available free energy to drive most of the familiar irreversible processes in nature.

from the cosmic microwave background radiation (CMB), which has travelled undisturbed from a time about 380 000 years after the Big Bang, and carries an imprint of the state of the universe at that relatively early time. Famously, the CMB carries the distinctive hallmark of thermodynamic equilibrium: its spectrum traces precisely that of black-body radiation (see figure 3). This implies that, at 380 000 years at least, the matter and radiation in the universe was in a state very close to the maximum entropy one of thermodynamic equilibrium. So what is wrong?

The solution to this conundrum lies with the expansion of the universe. The second law of thermodynamics, remember, applies to a *closed* system. The cosmological matter and radiation do not constitute a closed system when they are subject to the expansion of the universe. In effect, the expansion pulls the matter and radiation out of equilibrium. The full story of this is complicated and has to do with such processes as the nuclear reactions that took place during the first three minutes (Davies 1974, Albrecht 2004). Here I will limit my explanation to a simpler example. Imagine that the universe is filled uniformly with a mixture of a non-relativistic fluid (dust) and radiation, initially at a common temperature. It is a well-known result that as the universe expands the temperatures of these two components fall like $1/a^2$ and $1/a$ respectively, where a is the cosmological scale factor. Thus, in the absence of strong coupling between the matter and radiation, a temperature difference opens up between them. In other words, what started out as thermodynamic equilibrium is driven, by the expansion, to a state of non-equilibrium. If the matter and radiation are allowed to equilibrate by coupling, then the entropy will rise as a result of the irreversible transfer of heat energy from the hot radiation to the cooler matter.

A simplified and schematic illustration of the entropic history of the universe is shown in figure 4. The red curve represents the maximum permissible entropy of a co-moving region of the universe, which rises as the universe expands. The green curve represents the *actual* entropy of this region. Near the beginning (say, at 380 000 years, at the time of decoupling of matter and radiation) the green and red curves coincided, implying a condition of (temporary) thermodynamic equilibrium, as confirmed by the spectrum of the CMB. But as the universe expanded, a gap opened up between the two curves, with the actual entropy falling behind the maximum entropy permissible at that time (and at that value of a). It is this entropy gap that permits all the entropy-generating processes (such as star burning) that provide the thermodynamic arrow of time – processes that are trying to close the gap. Life itself feeds off that gap, so we owe our very existence to the effects of differential growth in the two entropy curves. (In practice, the biggest gap between the two curves opened up, not after decoupling, but during the era of nucleosynthesis, but that is another story [Davies 1974].) The question remains of whether the entropy gap will ever close, leading to the infamous heat death, or whether, in the context of an expanding universe, it may be arbitrarily delayed (Davies 1994).

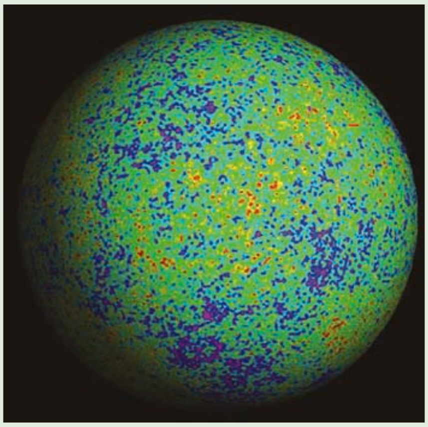
Gravitation changes the story

The cosmological expansion is a manifestation of the *gravitational* activity of the universe. Uniform expansion is, of course, an oversimplified description of the cosmological gravitational field. A more careful inspection of the CMB data, as provided by WMAP, reveals all-important fluctuations in temperature and density at a level of about one part in 10^5. These ripples in the CMB are the seeds of the large-scale structure of the universe, features that were destined to grow into clusters of galaxies (figure 5). The mechanism of the growth of clumpiness is gravitational aggregation. Regions of the universe that were slightly over-dense had a tendency to pull in material at the expense of their surroundings and thus amplify the density contrast. This tendency for gravitation to turn smooth distributions of gas into clumpy ones stands in stark contrast to the tendency of gases for which gravitational effects are negligible to go from clumpy to smooth (as in figure 1).

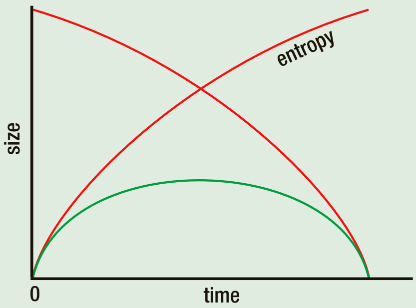
Loosely speaking, then, a clumpy laboratory gas represents a low-entropy state whereas a smooth gas represents a high-entropy state. For a gravitating system it is the other way around. The end state of this amplification of gravitational clumping is the black hole, which may be regarded as the maximum entropy equilibrium state of gravitating matter. This leads us to draw a significant conclusion about the early universe. Whereas the matter content of the universe was in a state close to thermodynamic equilibrium at early times, the gravitational field was very far from equilibrium.

How far? A way of quantifying the gap has been given by Penrose (1979, 2004) by appealing to the Bekenstein–Hawking formula for the entropy of a black hole (Bekenstein 1973, Hawking 1975). Taking the contents of the entire observable universe today, it is straightforward to compute the entropy of a black hole of equivalent mass. The answer is $10^{123}k$ where k is Boltzmann's constant. Compare this with the actual entropy derived from the CMB of about $10^{90}k$ at the time of decoupling, or $10^{100}k$ today. Clearly the universe was – and still is – very far indeed from the maximum possible entropy state. This mismatch becomes even more striking when translated into probabilities. The entropy of a state is logarithmically related to the number of microstates that can make it up, which means that the probability of a given state of less-than-maximum entropy declines exponentially with the entropy. If we ask what is the probability that a randomly chosen initial state of the universe would possess an entropy of only $10^{90}k$ rather than $10^{123}k$ and the answer comes out about $\exp(-10^{123})$, a number of mind-boggling smallness.

How can we explain this result? To recap, the arrow of time derives ultimately from the fact that the universe began in an exceedingly low-entropy (smooth) gravitational state, with almost all the gravitational activity concentrated in a single, orderly, dilatory mode, and only slight irregularities superimposed on this. This initial state was therefore, from the gravitational point of view, exceedingly special and remarkable, yet an essential element in explaining the universe we perceive. Do we just leave it at that,



5: Data from WMAP show small fluctuations in the otherwise smooth cosmic microwave background radiation. The slightly over-dense regions (shown in red) were destined to grow into clusters of galaxies under the action of gravitational clumping. (Courtesy NASA)



6: The green line shows the scale factor *a* as a function of time in a recontracting universe. The cosmological motion is approximately time symmetric. The application of quantum mechanics to the universe as a whole – a speculative exercise – suggests that, with very high probability, the entropy does not mimic the behaviour of *a*. Rather, the entropy either starts low and ends high, or vice versa (red lines). Adopting the fashionable "many universes" or "parallel universes" interpretation of quantum mechanics, we may conclude that the overall wave function of the universe has no preferred time orientation, although most individual branches of the wave function – considered to be really existing universes in this interpretation – do possess a thermodynamic arrow of time.

and accept that the universe was born in an exceedingly unusual state? Or is there a deeper explanation?

Inflation the answer?

Today there is a ready explanation for the initial smoothness of the universe. It is called inflation. According to the fashionable inflationary scenario (see, for example, Greene 2004), the universe jumped in size by an enormous factor during the first split second of its existence. This had the effect of "smoothing the crinkles" that may have been present initially by vastly distending space. The fleeting episode of accelerating expansion was driven by the excitation of a hypothesized "inflaton" field, which possessed negative pressure leading to a form of antigravity. The details are not important here. The point I want to make is that the temptation to appeal to an earlier physical process to explain the "initial" conditions of the universe merely passes the buck one more step. The equations governing the inflaton field are time-symmetric like all the others and one may still ask: how did the inflaton field get into its low-entropy excited state in the first place? In other words, one merely shifts the specialness of the initial gravitational state to the specialness of the inflaton field.

At the end of the day, it will never be possible to derive asymmetry from symmetry, and one seems to be left with the following alternatives:

- Accept a special initial state as a brute fact.
- Postulate some form of time-asymmetric law, possibly restricted in its major effects to the very early universe. Penrose has suggested this (Penrose 2004).
- Use some sort of anthropic argument as a selection effect, along the following lines. Only in regions where the inflaton field, by accident, happened to be in a special excited state initially would the universe inflate and achieve a high degree of gravitational uniformity, from which galaxies, stars and eventually life would emerge in an orderly manner. Observers will find themselves only in those very atypical regions where this state of affairs pertains. Most of the universe will be characterized by gravitationally very clumpy states, with large black holes, highly chaotic cosmological expansion, etc, which is hostile to life and goes unobserved.
- Appeal to quantum mechanics.

Quantum mechanics

The latter is my personal favourite, and I should like to close by briefly outlining how it works. Quantum mechanics may be applied to the entire universe in the manner discussed by Hartle and Hawking (1983), by taking the wave function of the universe to be a sum over all possible cosmic histories and geometries. The question of how to interpret such a wave function has been much discussed, but the consensus seems to be that only the so-called many universes interpretation makes sense. That is, one supposes that every branch of the cosmic wave function describes a really existing universe, and that this infinity of universes co-exist in parallel (Everett 1957, Deutsch 1996). So this is one variety of a theory of all possible worlds.

The implications are easiest to understand if we restrict to the wave function for all possible recontracting universes only. These are universes that start out with a big bang and end with a big crunch (see figure 6). The gross dynamical behaviour is thus symmetric in time, on average. However, in individual universes (i.e. branches of the wave function) the entropy will not follow the cosmic scale factor by rising and falling again. Rather, it will most probably start out low and end up high, or vice versa, as shown in figure 6. Any inhabitants of these universes would by definition label the low-entropy end "the big bang" and the high-entropy end "the big crunch", so from their point of view it doesn't matter which way round it is for any particular branch of the wave function (i.e. universe). The key thing is that *for the assemblage of universes as a whole* the situation remains time symmetric. (In technical parlance, the wave function is unitary.) So we embed special universes with time asymmetry in an overall time-symmetric ensemble. Note that in very rare cases (since the wave function of the universe contains all possible recontracting universes) the entropy will rise and then fall again, allowing the arrow of time to reverse (sometimes mis-stated as "time running backwards"). But the probability of our inhabiting such a universe is infinitesimal.

It seems to me that this is the best we can do to both have our cake and eat it. It accounts for the existence of an arrow of time without imposing one via ad hoc special laws, and without making undue appeal to anthropic selection. Its weakness lies with the fact that applying quantum mechanics to the entire universe remains a highly speculative exercise, and interpreting the wave function in the manner I have described is far from generally accepted. ●

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